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A New Interpretation of Bell's Inequalities

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Bell's inequalities are always derived assuming that local hidden-variable theories give a set of positive-definite probabilities for detecting a particle with a given spin orientation. The usual claim is that quantum mechanics, by its very nature, cannot produce a set of such probabilities. We show that this is not the case if one allows for generalized (nonpositive-definite) "master probability distributions." The master distributions give the usual quantum mechanical violation of Bell's inequalities. Consequences for the interpretation of quantum mechanics are discussed.

1. INTRODUCTION

Since their appearance, Bell's (1964) inequalities have been the subject of hundreds, if not thousands, of papers. Certainly it can be said that Bell's (1964) work has been the most influential on the interpretation of quantum mechanics since the famous argument of Einstein, Podolsky, and Rosen (1935).

As is well known, Einstein, Podolsky, and Rosen (EPR) criticized quantum mechanics as being an incomplete description of the world because it violated the so-called "criterion of physical reality." The criterion of physical reality, in turn, relied on the special-relativistic assumption that no signal can propagate faster than the speed of light: no two regions that are separated from each other by a spacelike interval can causally influence one another. This assumption is usually referred to as the "locality condition." Because in the quantum mechanical thought experiment envisaged by EPR (see next section) an observer in one region can apparently determine the outcome of an experiment in a second region that is separated from it by a spacelike interval, then either the locality condition is incorrect or quantum mechanics is not a complete description of reality.

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If one takes the stand that the locality condition must remain firm (as did EPR), then the presence of "local hidden variables," which must be added to quantum mechanics to make it complete, becomes a real possibility. Bell's contribution was to show that the locality condition leads to a set of definite mathematical predictions (the Bell inequalities) which violate the predictions of quantum mechanics. Experiments have repeatedly upheld quantum mechanics, thereby implying that the locality condition is incorrect.

That no two causally disjoint regions can influence each other is a principle firmly embedded in the consciousness of most physicists. It is therefore natural that Bell's theorem has received many elaborations in the literature and that many words have been devoted to its implications. As representative of the words we may take those found in the review by Clauser and Shimony (1978): "The conclusions [of Bell's theorem] are philosophically startling: either one must totally abandon the realistic philosophy of most working scientists, or dramatically revise our concept of spacetime."

In this article we propose another "out." If one examines the various proofs of Bell's theorem [see Clauser and Shimony (1978) for a partial list] the locality condition appears in various guises: the assumption of "philosophical realism" (Clauser and Shimony, 1973), "local realism" (Braunstein and Davies, 1989), "counterfactual definiteness" (Clauser and Shimony, 1978; Stapp, 1979), rejection of "action at a distance" (Clauser and Shimony, 1973), "principle of separability" (d'Espagnat, 1976), and the like. Mathematically, however, the crucial assumption in all the proofs is that the locality condition introduces a certain set of *a priori*, positive-definite probabilities P that are not predicted by quantum mechanics. [At first sight, Stapp's (1979) and (1985) proofs appear not to rely on any statement of probabilities. However, the equivalence theorem cited in Section 3.8 of Clauser and Shimony (1978) shows that Stapp's assumptions are equivalent to the assumption of the probabilities P.]

We show in Section 3 that, if one relaxes the requirement that the probabilities be positive definite, quantum mechanics indeed predicts a set of "*a priori*" probabilities. These probabilities are meaningful in the sense that they give the standard quantum mechanical results in physical circumstances, including the usual violation of Bell's inequalities. If one chooses to accept these probabilities as meaningful, then the probabilities arising from the assumption of hidden variables can be regarded as merely the "wrong answer" to a quantum mechanical problem, i.e., as merely the result of choosing the wrong set of probabilities. We illustrate these concepts first for a spin-1/2 system.

2. SPIN 1/2

As already mentioned, many versions of Bell's theorem have appeared in the literature. The simplest we have seen is Wigner's model as presented by Sakurai (1985). For our purposes it is sufficient to follow this proof; the reader is referred to Sakurai's text for further details.

We consider the usual spin-singlet system due to Bohm (1951) in which two spin-1/2 particles with total angular momentum equal to zero each pass through a Stern–Gerlach-type detector that is separated from a second identical detector by a spacelike interval. We also assume that measurements of the spin of each particle may be made along three unit vectors **a**, **b**, **c**, such that if the first detector records $\mathbf{S}_1 \cdot \mathbf{a}$ as "up" (where $\mathbf{S}_1$ refers to the spin of the first particle), then the second detector must record $\mathbf{S}_2 \cdot \mathbf{a}$ as "down." We denote up and down by $+$ and $-$, respectively.

Quantum mechanics asserts that each particle emitted from a source has no definite spin but is in a superposition of states. The hidden variables approach assumes, on the contrary, that the particles have a definite spin. For instance, a certain fraction of the particles measured by detector 1 will be of the type $(\mathbf{a}+, \mathbf{b}-, \mathbf{c}-)$. (The detector, of course, will only measure the spin along one axis.) It is not important what fraction of the particles are of this type; one merely requires that some definite fraction manifest spins in those directions. To ensure zero total angular momentum, when detector 1 measures a particle of the above type, detector 2 must measure a particle of type $(\mathbf{a}-, \mathbf{b}+, \mathbf{c}+)$. There will be eight such spin combinations (see Table I), so the number of each particle type may be labeled $N_1 \ldots N_8$.

Assuming the N to be nonnegative, they must satisfy inequalities of the form:

$$N_3 + N_5 \leqslant (N_2 + N_5) + (N_3 + N_7) \tag{1}$$

From Table I one sees that $N_3 + N_5$ represents the total number of particle pairs for which detector 1 measures $\mathbf{S}_1 \cdot \mathbf{a}$ to be $+$ and detector 2 measures $\mathbf{S}_2 \cdot \mathbf{b}$ to be $+$, etc. If $P(\mathbf{a}+, \mathbf{b}+)$ denotes the probability that $(\mathbf{S}_1 \cdot \mathbf{a} = +,\ \mathbf{S}_2 \cdot \mathbf{b} = +)$, then clearly

$$P(\mathbf{a}+, \mathbf{b}+) = \frac{N_3 + N_5}{\sum_{i=1}^{8} N_i} \tag{2}$$

with similar expressions for the other pairwise probabilities. Then equation (1) becomes

$$P(a+, b+) \leqslant P(\mathbf{a}+, \mathbf{c}+) + P(\mathbf{c}+, \mathbf{b}+) \tag{3}$$

Table I. All Eight Combinations of Spin-1/2 Particle Types ($\mathbf{a}\pm, \mathbf{b}\pm, \mathbf{c}\pm$) Emitted from a Source According to the Hidden-Variable Model[a]

Population	Particle 1	Particle 2	QM Probability
N_1	$(+++)$	$(---)$	$\frac{1}{4}\cos\alpha\,(1+\cos\alpha)$
N_2	$(++-)$	$(--+)$	$\frac{1}{4}\cos\alpha\,(-1+\cos\alpha)$
N_3	$(+-+)$	$(-+-)$	$\frac{1}{4}\sin^2\alpha$
N_4	$(-++)$	$(+--)$	$\frac{1}{4}\sin^2\alpha$
N_5	$(+--)$	$(-++)$	$\frac{1}{4}\sin^2\alpha$
N_6	$(-+-)$	$(+-+)$	$\frac{1}{4}\sin^2\alpha$
N_7	$(--+)$	$(++-)$	$\frac{1}{4}\cos\alpha\,(-1+\cos\alpha)$
N_8	$(---)$	$(+++)$	$\frac{1}{4}\cos\alpha\,(1+\cos\alpha)$

[a] The hidden-variable model assumes spin-1/2 particles of type ($\mathbf{a}\pm, \mathbf{b}\pm, \mathbf{c}\pm$) are emitted from a source. The table lists all eight combinations of ($\mathbf{a}\pm, \mathbf{b}\pm, \mathbf{c}\pm$). The notation $(++-)$, etc., means that the spin along the **a** and **b** axes is measured to be $+$, but along the **c** axis, $-$. To ensure zero total angular momentum, a particle 1 of the type $(++-)$ must be accompanied by a particle 2 of the type $(--+)$. The last column lists the calculated quantum mechanical probability for measuring a given particle type.

which is one of the Bell inequalities. Note that the *inequality* results from the assumption that the P's are *nonnegative.*

It is easy to show that these inequalities violate the assumptions of quantum mechanics. Let us write the quantum mechanical projection operator in the form (Schiff, 1968)

$$\Pi(\mathbf{a}\pm) = \tfrac{1}{2}(1 \pm \boldsymbol{\sigma}\cdot\mathbf{a}) \tag{4}$$

where $\boldsymbol{\sigma}\cdot\mathbf{a}$ is a 2×2 matrix and $\boldsymbol{\sigma}$ represents the Pauli spin matrices. The quantum mechanical probability of finding particle 1 in the $+$ state is $\frac{1}{2}\operatorname{Tr}\Pi(\mathbf{a}) = 1/2$. Similarly, the joint probability $P(\mathbf{a}+, \mathbf{b}\pm)$ is

$$\begin{aligned} P(\mathbf{a}+, \mathbf{b}\pm) &= \tfrac{1}{2}\operatorname{Tr}\{\Pi(\mathbf{a})\,\Pi(\mathbf{b}\pm)\} \\ &= \tfrac{1}{4}(1 \pm \mathbf{a}\cdot\mathbf{b}) \end{aligned} \tag{5}$$

with similar expressions for the other joint probabilities.

If, for simplicity, we choose **a**, **b**, **c** to lie in a plane with $\mathbf{a}\cdot\mathbf{c} = \mathbf{b}\cdot\mathbf{c} = \cos 2\alpha$, $\mathbf{a}\cdot\mathbf{b} = \cos 4\alpha$, then equation (3) becomes

$$\sin^2 2\alpha \leqslant 2\sin^2\alpha \tag{6}$$

which is in general *not true.* Thus, quantum mechanics violates Bell's inequalities.

3. QUANTUM MECHANICAL GENERALIZATION

Everything we have done thus far is standard. The usual statement is the one mentioned above: the difference between the hidden variables approach and quantum mechanics is that a local hidden variables theory assumes that the particles' spin states have some definite probability, such as given by equation (2), whereas quantum mechanics makes no such prediction—a given particle's spin is in a superposition of states. We now demonstrate that this is not entirely the case.

Let us form probabilities of the type $P(\mathbf{a}\lambda, \mathbf{b}\mu, \mathbf{c}\nu)$, where $\lambda, \mu, \nu = \pm 1$ in conformity with our previous notation. These joint probabilities represent correlations between three "simultaneous" measurements on two particles. Of course, this is not physically possible in the systems we have been discussing; however, this objection will be answered below.

There are several possible rules for forming the probabilities P. For instance, we may write

$$P(\mathbf{a}\lambda, \mathbf{b}\mu, \mathbf{c}\nu) = \frac{1}{2} \operatorname{Tr}\{\Pi(\mathbf{a})\, \Pi(\mathbf{b})\, \Pi(\mathbf{c})\} \tag{7a}$$

or

$$P(\mathbf{a}\lambda, \mathbf{b}\mu, \mathbf{c}\nu) = \frac{1}{4} \operatorname{Tr}\{\Pi(\mathbf{a})\, \Pi(\mathbf{b})\, \Pi(\mathbf{c}) + \Pi(\mathbf{c})\, \Pi(\mathbf{b})\, \Pi(\mathbf{a})\} \tag{7b}$$

or

$$P(\mathbf{a}\lambda, \mathbf{b}\mu, \mathbf{c}\nu) = \frac{1}{3!} \operatorname{Tr}\{\Pi(\mathbf{a})\, \Pi(\mathbf{b})\, \Pi(\mathbf{c}) + \text{permutations}\} \tag{7c}$$

Equation (7a) gives a complex result due to the complex off-diagonal elements in the Pauli matrix $\boldsymbol{\sigma}_y$. This problem is eliminated by (7b), although the sum is not symmetric in the arguments. The most natural generalization of equation (5) is (7c), which is both symmetric in the arguments and gives a real result for P. (In fact, it is not important which version one chooses; when one sums over the third argument, as will be done below, the result is always real.)

Choosing (7c) as the algorithm for computing the triple probabilies, we get

$$P(\mathbf{a}\lambda, \mathbf{b}\mu, \mathbf{c}\nu) = \tfrac{1}{8}[1 + \lambda\mu\mathbf{a}\cdot\mathbf{b} + \lambda\nu\mathbf{a}\cdot\mathbf{c} + \mu\nu\mathbf{b}\cdot\mathbf{c}] \tag{8}$$

Working this out with $\mathbf{a}\cdot\mathbf{c} = \mathbf{b}\cdot\mathbf{c} = \cos 2\alpha$ and $\mathbf{a}\cdot\mathbf{b} = \cos 4\alpha$ as before gives, in obvious notation,

$$P(+++) = P(---) = \tfrac{1}{4}\cos\alpha\,(1+\cos\alpha)$$
$$P(++-) = P(--+) = \tfrac{1}{4}\cos\alpha\,(-1+\cos\alpha) \qquad (9)$$

$$P(+-+) = P(-+-) = \tfrac{1}{4}\sin^2\alpha$$
$$P(+--) = P(-++) = \tfrac{1}{4}\sin^2\alpha \qquad (10)$$

For convenience we list these probabilities in the last column in Table I. The first thing one notices about the above P is that two of them can become negative. This might cause one to reject them as nonphysical. We discuss this matter in more detail below. For now we note that the eight P do sum to 1.

As before, one wants for the EPR experiment the *pairwise* probabilities $P(\mathbf{a}, \mathbf{b})$, etc. To get them we add the $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$'s from Table I that correspond to the previous N_i; then the Bell inequality (3) becomes

$$0 \leqslant \tfrac{1}{2}\cos\alpha\,(-1+\cos\alpha) \qquad (11)$$

This addition has summed over the third argument in the new $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$'s and reduced them to $P(\mathbf{a}, \mathbf{b})$'s. With the half-angle formula $2\sin^2(\alpha/2) = 1-\cos\alpha$, equation (10) reduces to the usual quantum mechanical result (6).

Thus we see that quantum mechanics does indeed predict a set of probabilities (9) in exactly the same way as do hidden variable theories. When the summation is carried over the third argument, the usual quantum mechanical relationships for two-particle correlations are recovered. Due to the fact that the third argument disappears in this process, the "nonphysical nature" of the $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$ becomes irrelevant.

One might object that the result depends on which of the rules (7) is chosen to form the P's. However, once the sum over the third argument is carried out, all the rules give the same answer. For example, the imaginary term in $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$ resulting from rule (7a) is $\mathrm{Tr}\,\lambda\mu\nu(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b})(\boldsymbol{\sigma}\cdot\mathbf{c})$. When all the P's for Table I are computed and summed for the Bell inequality, all these terms cancel out pairwise due to the permutations of sign $(\lambda\mu\nu)$.

Consequently, nothing we have done is outside the domain of quantum mechanics; the only initially strange aspect of the entire procedure is that the P's are not positive definite.

4. SPIN-1 CASE

A second objection that might be raised is that the procedure just outlined only works for the spin-1/2 case. This is not true. We shall now demonstrate the procedure for spin 1, a procedure that can be generalized to arbitrary spin.

Unlike the spin-1/2 case, where there were eight possible spin combinations, the spin-1 case has 27, here listed in Table II. The possibility of a zero spin projection on the **a**, **b**, **c** axes precludes the writing of the usual Bell inequalities. For instance, if we formed the inequality (3) with values from Table II, we would get

$$N_3 + N_5 + N_{23} \overset{?}{\leqslant} N_2 + N_5 + N_{22} + N_3 + N_7 + N_{25} \tag{12}$$

which is not generally valid. (Note that this has nothing to do with quantum mechanics or hidden variables; it is a mere result of combinatorics.)

Triangle-type inequalities that hold for the spin-1 case, however, can be found. If we define

$$\begin{aligned} P(\mathbf{a}, \mathbf{b}) \equiv{} & P(\mathbf{S}_1 \cdot \mathbf{a} = +, \mathbf{S}_2 \cdot \mathbf{b} = +) + P(\mathbf{S}_1 \cdot \mathbf{a} = +, \mathbf{S}_2 \cdot \mathbf{b} = 0) \\ & + P(\mathbf{S}_1 \cdot \mathbf{a} = 0, \mathbf{S}_2 \cdot \mathbf{b} = -) \end{aligned} \tag{13}$$

with analogous expressions for $P(\mathbf{a}, \mathbf{c})$ and $P(\mathbf{a}, \mathbf{b})$, then valid extensions of the Bell inequalities are

$$P(\mathbf{a}, \mathbf{b}) \leqslant P(\mathbf{b}, \mathbf{c}) + P(\mathbf{c}, \mathbf{a}) \tag{14a}$$

$$P(\mathbf{b}, \mathbf{c}) \leqslant P(\mathbf{a}, \mathbf{b}) + P(\mathbf{c}, \mathbf{a}) \tag{14b}$$

$$P(\mathbf{c}, \mathbf{a}) \leqslant P(\mathbf{a}, \mathbf{b}) + P(\mathbf{b}, \mathbf{c}) \tag{14c}$$

Note that since each of the P's contains three terms, these inequalities each involve nine of the terms in Table II. Furthermore, to obtain these expressions one must make use of the fact that angular momentum is conserved. For instance, (14a) reduces to the expression

$$P(-+-) + P(+--) \leqslant P(-++) + P(+-+) + (\text{other } P\text{'s})$$

However, to ensure zero total angular momentum, $P(-+-) = P(+-+)$ and $P(-++) = P(+--)$. Thus the inequality is valid.

To check whether the inequalities (14) are violated by quantum mechanics, one must now compute the 27 quantum mechanical probabilities associated with the 27 N_i. This is done by the same method of Section 3. For spin-1 particles, however, the projection operators are formed from the 3×3 angular momentum matrices S analogous to the Pauli matrices (Schiff, 1968). In this case the projection operators can be written

$$\Pi(\mathbf{a}+) = \tfrac{1}{2}\mathbf{S} \cdot \mathbf{a}(\mathbf{1} + \mathbf{S} \cdot \mathbf{a}) \tag{15a}$$

$$\Pi(\mathbf{a}-) = \tfrac{1}{2}\mathbf{S} \cdot \mathbf{a}(\mathbf{S} \cdot \mathbf{a} - \mathbf{1}) \tag{15b}$$

$$\Pi(\mathbf{a}0) = 1 - (\mathbf{S} \cdot \mathbf{a})^2 \tag{15c}$$

Table II. Combinations of Spin-1 Particles Emitted from a Source According to the Hidden-Variable Model[a]

Population	Particle 1	Particle 2	QM Probability
N_1	$(+++)$	$(---)$	$\frac{1}{24}\{A+B+C+A^2+B^2+C^2 + AB+AC+BC-ABC\}$
N_2	$(++-)$	$(--+)$	$\frac{1}{24}\{A-B-C+A^2+B^2+C^2 -AB-AC+BC-ABC\}$
N_3	$(+-+)$	$(-+-)$	$\frac{1}{24}\{-A-B+C+A^2+B^2+C^2 +AB-AC-BC-ABC\}$
N_4	$(-++)$	$(+--)$	$\frac{1}{24}\{-A+B-C+A^2+B^2+C^2 -AB+AC-BC-ABC\}$
N_5	$(+--)$	$(-++)$	$\frac{1}{24}\{-A+B-C+A^2+B^2+C^2 -AB+AC-BC-ABC\}$
N_6	$(-+-)$	$(+-+)$	$\frac{1}{24}\{-A-B+C+A^2+B^2+C^2 +AB-AC-BC-ABC\}$
N_7	$(--+)$	$(++-)$	$\frac{1}{24}\{A-B-C+A^2+B^2+C^2 -AB-AC+BC-ABC\}$
N_8	$(---)$	$(+++)$	$\frac{1}{24}\{A+B+C+A^2+B^2+C^2 +AB+AC+BC-ABC\}$
N_9	$(++0)$	$(--0)$	$\frac{1}{12}\{1+A-B^2-C^2-BC+ABC\}$
N_{10}	$(+0+)$	$(-0-)$	$\frac{1}{12}\{1+C-A^2-B^2-AB+ABC\}$
N_{11}	$(0++)$	$(0--)$	$\frac{1}{12}\{1+B-A^2-C^2-AC+ABC\}$
N_{12}	$(+00)$	(-00)	$\frac{1}{6}\{B^2-ABC\}$
N_{13}	$(0+0)$	$(0-0)$	$\frac{1}{6}\{C^2-ABC\}$
N_{14}	$(00+)$	$(00-)$	$\frac{1}{6}\{A^2-ABC\}$
N_{15}	(000)	(000)	$\frac{1}{3}\{ABC\}$
N_{16}	$(00-)$	$(00+)$	$\frac{1}{6}\{A^2-ABC\}$
N_{17}	$(0-0)$	$(0+0)$	$\frac{1}{6}\{C^2-ABC\}$
N_{18}	(-00)	$(+00)$	$\frac{1}{6}\{B^2-ABC\}$
N_{19}	$(0--)$	$(0++)$	$\frac{1}{12}\{1+B-A^2-C^2-AC+ABC\}$
N_{20}	$(-0-)$	$(+0+)$	$\frac{1}{12}\{1+C-A^2-B^2-AB+ABC\}$
N_{21}	$(--0)$	$(++0)$	$\frac{1}{12}\{1+A-B^2-C^2-BC+ABC\}$
N_{22}	$(+0+)$	$(-0+)$	$\frac{1}{12}\{1-C-A^2-B^2+AB+ABC\}$
N_{23}	$(+-0)$	$(-+0)$	$\frac{1}{12}\{1-A-B^2-C^2+BC+ABC\}$
N_{24}	$(0+-)$	$(0-+)$	$\frac{1}{12}\{1-B-A^2-C^2+AC+ABC\}$
N_{25}	$(0-+)$	$(0+-)$	$\frac{1}{12}\{1-B-A^2-C^2+AC+ABC\}$
N_{26}	$(-0+)$	$(+0-)$	$\frac{1}{12}\{1-C-A^2-B^2+AB+ABC\}$
N_{27}	$(-+0)$	$(+-0)$	$\frac{1}{12}\{1-A-B^2-C^2+BC+ABC\}$

[a]The same as Table I, except for spin-1 particles. In this case, +, −, or 0 spin may be measured along any of the axes and there are 27 possible combinations. We use the following shorthand: $A \equiv \mathbf{a}\cdot\mathbf{b}$; $B \equiv \mathbf{b}\cdot\mathbf{c}$; $C \equiv \mathbf{a}\cdot\mathbf{c}$.

The probabilities are computed by rule (7a) (here there are no complex terms) and are listed in the last column of Table II. Finally, assuming the same geometrical configuration as before, summing over the third argument gives the Bell inequality (13a), which reduces to

$$0 \leqslant 6 \cos^4 \alpha - 7 \cos^2 \alpha - 2 \cos \alpha + 3 \tag{16}$$

This expression is violated for values of $\cos \alpha$ above about 0.611.

5. INTERPRETATION AND CONCLUSIONS

We have shown that, although hidden variable theories predict a set of definite probabilities for spin measurements in an EPR-type experiment, quantum mechanics does also. Moreover, the probabilities it predicts are unambiguous because the rules available for calculating the probabilities all give the same final result; this result violates Bell's inequalities.

The obvious difference in the two procedures is that ours produces $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$'s that are non positive definite. This, however, seems to us more a semantic difficulty than a real one. To actually measure the $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$ would require three "simultaneous" measurements, which is not possible in a system with two particles when only one measurement on each particle is allowed. The P's should perhaps not be termed probabilities, but rather "inferred probabilities" or "master distributions" from which the actual pairwise correlations are obtained. Since the $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$ are not what is actually being measured, one should not worry about whether they are positive definite or not as long as they lead to physically valid predictions for measured quantities.

By the same token, the standard derivation of Bell's inequalities (Section 2) also assumes a $P(\mathbf{a}, \mathbf{b}, \mathbf{c})$ and the correlations must again be taken pairwise.

Thus what we have done is completely analogous to the hidden variable case.

Given the exact analogy between the two procedures, quantum mechanics and hidden variable theories are placed on a more nearly equal footing. From this perspective, one could regard the usual hidden-variable result as merely the wrong answer to a quantum mechanical problem; it is not surprising that a set of non-quantum mechanical rules gives a non-quantum mechanical result. By the same token, the view that quantum mechanics represents a nonlocal theory while local hidden variables represents a local theory also loses much of its force. "True beauty," it is said, "is always fresh." Quantum mechanics is thus truly beautiful.

NOTE ADDED IN PROOF

Since submitting this work, we have discovered that others have considered negative probabilities as an "out" to the EPR paradox. (See Mückenheim, W., *et al.* (1986), *Physics Reports*, **133**, 337.)

Also, before rejecting negative probabilities out of hand, one should consider other "unreal" quantities in physics: $\sqrt{-1}$, imaginary time and the wave function ψ itself. Furthermore, there was no *a priori* reason negative probabilities should have made the EPR paradox vanish.

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